Intro to Multiple Time Series Assignment 1

Samet Efe Keskin
Erasmus+ Student from Bilkent University, at University of Mannheim

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1 About the Data Set and This Analysis

One can see that the dataset is quarterly US data on real GDP and real aggregate consumption for 1947Q1-2016Q4. To start with, I first check the stationarity. Though there are many ways to check, a visual inspection is good for the first glance.

2 Are GDP and Consumption Log Growth Rates Stationary?

By performing the first part of the Stata code, which has been uploaded as a separate file, I've acquired the following visual pattern.

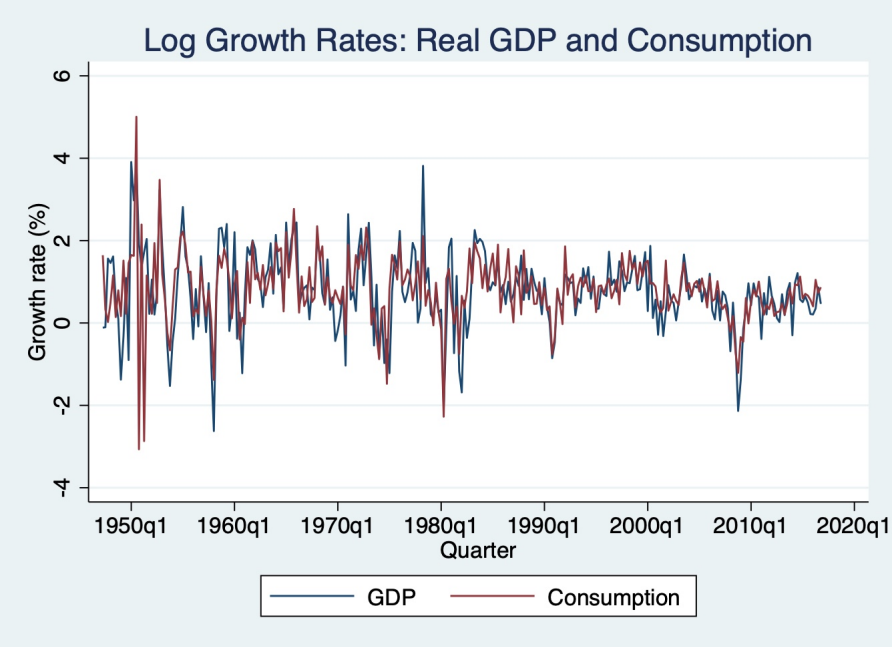

Figure 1: Example figure

The growth rate of GDP and consumption fluctuates around a constant mean and shows no apparent trends, which points to weak stationarity. Just to be formally sure, I proceeded with ADF tests. From

the appendix, you can see the test results, which have a very low p-value. I reject the unit root and, hence, formally accept stationary.

3 Optimal Lag Selection for potential AR Model

In this part, I detect the optimal leg number by Partial Autocorrelation Function (PACF) to measure the pure effect of a past lag on the current date. For instance, PACF at lag 1 displays the pure impact of value at t-1 on t. Here are the results:

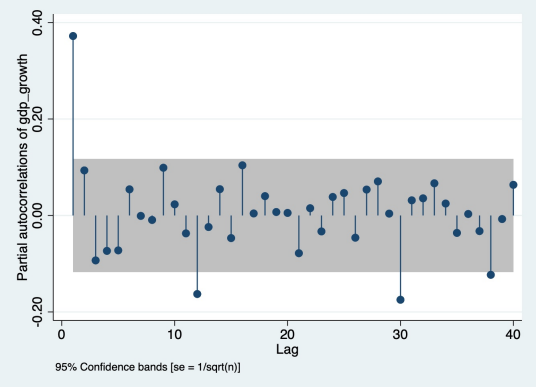

Figure 2: PAC of Real GDP Growth

The figure 2 shows a highly significant jump at lag 1. Nonetheless, the other lags show no promising results with respect to the shaded 95 per cent confidence interval. Hence, the optimal AR order for GDP growth may be AR(1).

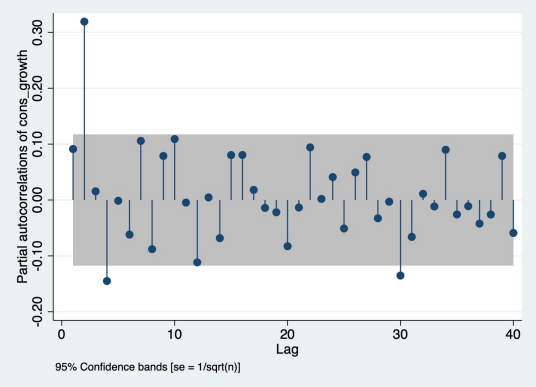

Figure 3: PAC of Real Consumption Growth

Figure 3 displays a substantially significant jump at lag 1 again. Even though lags 2, 3, and 4 show some oscillations, there are no statistically significant results. Hence, the optimal AR order for GDP growth may be AR(1) or AR(2), depending on the purpose of the study.

4 AR Models and OLS Estimation

Since the main question is, **"Does the past help explain the present?"** we formally answer it by running an Ordinary Least Squares estimation. In other words, we search for an answer to *whether previous lags influence the current GDP growth.*

$$\widehat{\text{gdp\_growth}}_t = 0.4894 + 0.3720 \cdot \text{gdp\_growth}_{t-1} \tag{1}$$

We estimate a first-order autoregressive model for GDP growth in Equation 1. The coefficient on the lagged GDP growth term is statistically significant and less than 1 in absolute value, indicating a stable AR process.GDP growth displays constancy, and the previous quarter's growth has a significant impact on expected growth at the current time.

$$\widehat{\text{cons\_growth}}_t = 0.5014 + 0.0645 \cdot \text{cons\_growth}_{t-1} + 0.3195 \cdot \text{cons\_growth}_{t-2} \tag{2}$$

Equation 2 presents the AR(2) model for consumption growth. Additionally, one might argue that consumption growth is largely affected by the flow of two quarters back, rather than one.

Now, I estimate the models using OLS. Here are the regressions:

```
. reg gdp_growth L.gdp_growth
```

Source	SS	df	MS	Number of obs	=	278
				F(1, 276)	=	44.48
Model	34.4892368	1	34.4892368	Prob > F	=	0.0000
Residual	214.019335	276	.775432374	R-squared	=	0.1388
				Adj R-squared	=	0.1357
Total	248.508572	277	.89714286	Root MSE	=	.88059

gdp_growth	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
gdp_growth						
L1.	.3720182	.055782	6.67	0.000	.2622059	.4818305
_cons	.4893687	.0682854	7.17	0.000	.3549423	.6237951

Figure 4: Regression Results for OLS AR(1): GDP GROWTH

```
. * Estimate AR(2) for Consumption growth

. reg cons_growth L.cons_growth L2.cons_growth
```

Source	SS	df	MS	Number of obs	=	277
				F(2, 274)	=	17.01
Model	20.2840085	2	10.1420042	Prob > F	=	0.0000
Residual	163.378824	274	.596273082	R-squared	=	0.1104
				Adj R-squared	=	0.1039
Total	183.662833	276	.665445047	Root MSE	=	.77219

cons_growth	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
cons_growth						
L1.	.0645197	.057182	1.13	0.260	-.0480522	.1770917
L2.	.3193501	.0570759	5.60	0.000	.2069871	.4317131
_cons	.5013718	.0779852	6.43	0.000	.3478455	.6548982

Figure 5: Regression results for OLS AR(2): Consumption Growth

5 Cross-Autocorrelations and VAR Suitability

To evaluate whether a VAR model is preferable to two separate univariate AR models, one may estimate the cross-autocorrelations between the log growth rates of real GDP and real consumption.

I computed the following cross-correlations for lags $j = 1, 2, \ldots, 20$:

$$\text{corr}(\text{GDP}_t, \text{Cons}_{t-j}) \quad \text{and} \quad \text{corr}(\text{Cons}_t, \text{GDP}_{t-j}) \tag{3}$$

Below are the most relevant correlations for the first few lags:

Lag (j)	$\text{corr}(\text{GDP}_t, \text{Cons}_{t-j})$	$\text{corr}(\text{Cons}_t, \text{GDP}_{t-j})$
1	0.4689	0.3371
2	0.2969	0.1714
3	0.1175	0.0929
4	0.0877	-0.0614
5+	Small or negative	Small or inconsistent

Table 1: Cross-correlations between GDP and Consumption growth rates

From the table, we observe the following:

- $\text{corr}(\text{GDP}_t, \text{Cons}_{t-1}) = 0.4689$ and $\text{corr}(\text{Cons}_t, \text{GDP}_{t-1}) = 0.3371$ are both relatively strong.
- There is a strong positive correlation at lag 1 and moderately smaller at lag 2. So previous consumption growth has an information power over current GDP growth.
- This suggests that each series has predictive power for the other at short horizons.

Therefore, one can arguably state that:

- Past consumption affects the real GDP somehow.
- Past values of each variable help predict the other, especially at short lags.

Appendix: Regression Output and Correlation Results

A. Regression Estimates

A.1 AR(1) Model for GDP Growth

$$\widehat{\text{gdp\_growth}}_t = 0.4894 + 0.3720 \cdot \text{gdp\_growth}_{t-1} \tag{4}$$

$$\text{Standard Error} : (0.0683,\ 0.0558) \tag{5}$$

$$\text{P-values} : (0.000,\ 0.000) \tag{6}$$

$$R^2 = 0.1388, \quad \text{Adjusted } R^2 = 0.1357 \tag{7}$$

A.2 AR(2) Model for Consumption Growth

$$\widehat{\text{cons\_growth}}_t = 0.5014 + 0.0645 \cdot \text{cons\_growth}_{t-1} + 0.3195 \cdot \text{cons\_growth}_{t-2} \tag{8}$$

$$\text{Standard Errors} : (0.0779,\ 0.0572,\ 0.0571) \tag{9}$$

$$\text{P-values} : (0.000,\ 0.260,\ 0.000) \tag{10}$$

$$R^2 = 0.1104, \quad \text{Adjusted } R^2 = 0.1039 \tag{11}$$

B. Selected Cross-Autocorrelations

We report the estimated cross-correlations between GDP and consumption growth rates up to lag 4:

Lag (j)	$\text{corr}(\text{GDP}_t, \text{Cons}_{t-j})$	$\text{corr}(\text{Cons}_t, \text{GDP}_{t-j})$
1	0.4689	0.3371
2	0.2969	0.1714
3	0.1175	0.0929
4	0.0877	−0.0614

Table 2: Cross-correlations between GDP and Consumption Growth

Correlations beyond lag 4 were generally weak and inconsistent in sign, and are omitted for brevity.