Structural VAR Model and Permanent Income Hypothesis: Empirical Investigation

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1 About the Data Set and This Analysis

One can see that the dataset is quarterly US data on real GDP and real aggregate consumption for 1947Q1-2016Q4. To start with, I first check the stationarity. Though there are many ways to check, a visual inspection is good for the first glance.

2 VAR(4) Estimation and Stability Check

To examine the joint dynamics of GDP and consumption growth, I estimate a VAR(4) model using quarterly log-differenced GDP and consumption from 1948Q2 to 2016Q4. The model includes four lags of each variable in both equations.

The estimation output reveals that lagged consumption growth significantly predicts GDP growth, particularly at lags 1 and 2, while GDP growth also influences consumption through several lag terms. This suggests the presence of bi-directional Granger causality.

Table 1: Summary of VAR(4) Estimation Results

Equation	Parameters	RMSE	R-squared	p-value
GDP Growth (`dlgdp`)	9	0.00843	0.2332	¡ 0.001
Consumption Growth (`dlcons`)	9	0.00748	0.1871	¡ 0.001

To verify whether the VAR model is dynamically stable, I examine the modulus of the eigenvalues of the companion matrix. In Stata, this is done using the `varstable` command. The VAR model is considered stable if all eigenvalues lie strictly within the unit circle.

Table 2: Eigenvalue Moduli of the Companion Matrix

Eigenvalue (Real + Imag)	**Modulus**
$-0.5566 + 0.4582i$	0.7209
$-0.5566 - 0.4582i$	0.7209
$0.6056 + 0.2995i$	0.6756
$0.6056 - 0.2995i$	0.6756
$0.3364 + 0.3555i$	0.4894
$0.3364 - 0.3555i$	0.4894
-0.4020	0.4020
-0.2848	0.2848

As shown above, all eigenvalue moduli are well below 1. Thus, the VAR(4) model satisfies the stability condition. This ensures that the system is dynamically stable and that the response of each variable to shocks will converge rather than diverge over time. It also validates the use of impulse response functions in subsequent analysis.

Additionally, the model's information criteria support the chosen lag length: AIC $= -14.04$, BIC $= -13.80$, and HQIC $= -13.95$.

3 Residual Autocorrelation in the VAR Model

To evaluate whether the residuals from the VAR(4) model are white noise, I generate autocorrelation and partial autocorrelation plots for the residuals of both equations.

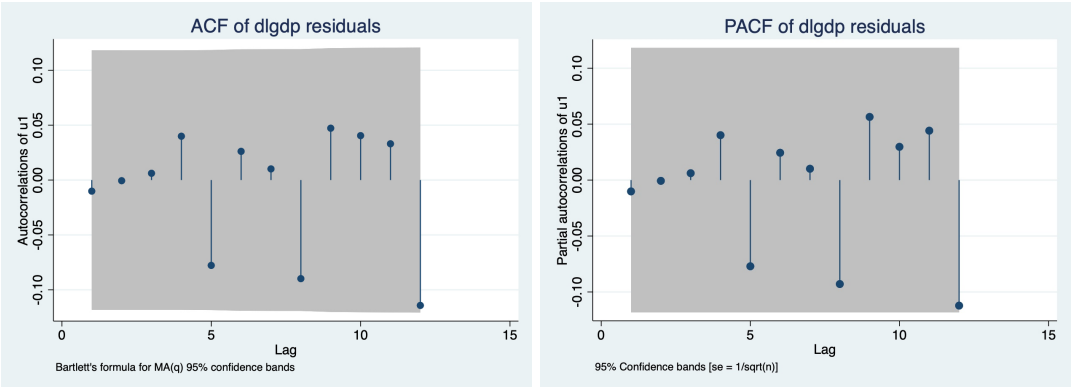

Figure 1: ACF and PACF of Residuals from GDP Growth Equation

As seen in Figure 1, the GDP growth residuals do not display significant autocorrelation. All spikes lie within the 95% confidence bands. This supports the assumption of white noise residuals in the first equation.

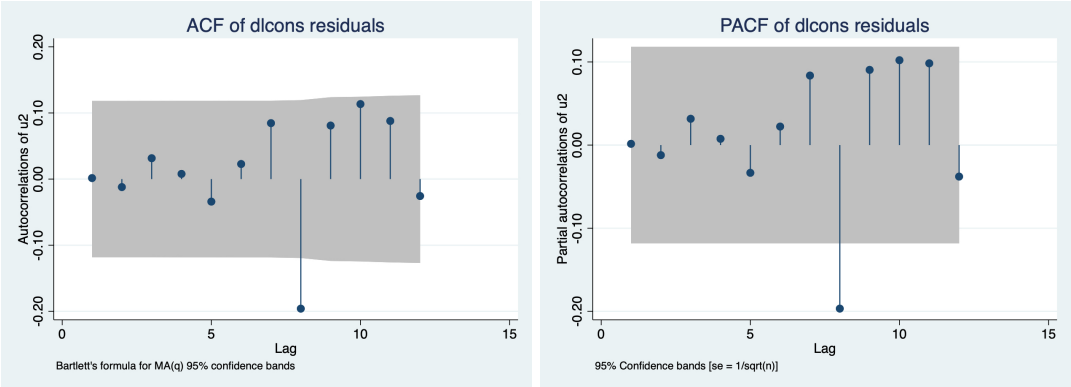

Figure 2: ACF and PACF of Residuals from Consumption Growth Equation

For the consumption equation, some moderate autocorrelation appears at higher lags (especially lags 8–11), as indicated by significant spikes outside the 95% bands. This suggests that the model may underfit certain dynamics in the consumption process. Further improvements could involve including additional lags or structural refinements.

Overall, the residuals from the VAR(4) system behave mostly like white noise, particularly for GDP growth, but there's room for improvement in modeling consumption dynamics more fully.

4 Structural Identification and Impulse Response Analysis

To distinguish between permanent and transitory income shocks, I estimate a Structural Vector Autoregression (SVAR) model based on the previously estimated VAR(4). The identification of structural shocks is motivated by the **Permanent Income Hypothesis** (PIH), which posits that rational consumers smooth consumption over time and primarily respond to permanent changes in income. This implies that consumption should *not* respond contemporaneously to transitory income shocks.

To translate this into an econometric model, I impose a short-run restriction: **a transitory income shock has no immediate effect on consumption**. Technically, I implement this by specifying a recursive identification structure (lower-triangular A matrix). In this structure:

- `dlgdp` (income growth) is ordered first,
- `dlcons` (consumption growth) is ordered second,
- The matrix A is constrained as:

$$A = \begin{bmatrix} 1 & 0 \\ a_{21} & 1 \end{bmatrix}$$

  where the zeros reflect the assumed contemporaneous exclusion restriction: consumption does not affect GDP contemporaneously.

Stata Implementation

In Stata, I manually defined this short-run structure as follows:

```
matrix A = (1, 0 \ ., 1)
svar dlgdp dlcons, aeq(A)
```

This syntax tells Stata to constrain $A[1,1] = 1$, $A[1,2] = 0$, and $A[2,2] = 1$, while leaving $A[2,1]$ (the contemporaneous effect of GDP on consumption) free to be estimated. The model converged after 3 iterations, with statistically significant parameters and a log likelihood of -509.1. The LR test of overidentifying restrictions yielded a test statistic of $\chi^2(2) = 4908$, with $p < 0.001$, confirming that the constraints are non-trivial and binding.

Impulse Response Functions (IRFs)

Using the estimated SVAR, I construct impulse response functions (IRFs) to trace the dynamic effects of a structural income shock on both GDP and consumption growth.

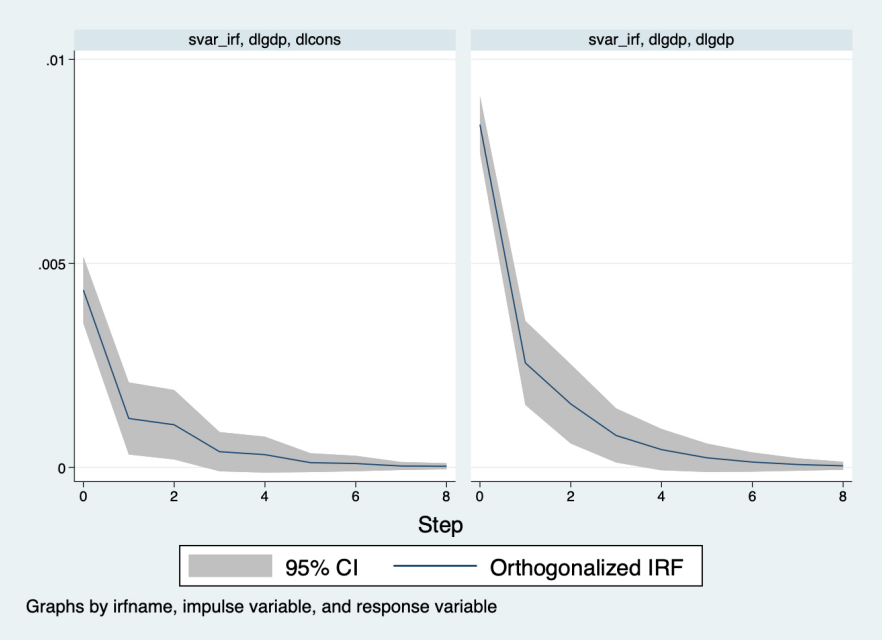

Figure 3: Orthogonalized Impulse Responses to a Structural Income Shock

Figure 3 shows the IRFs of GDP and consumption to a one-standard-deviation structural shock in income.

- **Right Panel (dlgdp → dlgdp)**: GDP growth responds immediately and strongly to its own shock. The effect decays over time, indicating that the income shock has both transitory and persistent components.
- **Left Panel (dlgdp → dlcons)**: Consumption growth exhibits *no impact* in period 0 — consistent with our identifying restriction. However, from period 1 onward, consumption adjusts gradually and positively, consistent with the view that it responds to the *perceived permanent component* of the income shock.

Conclusion

This empirical pattern aligns well with the permanent income hypothesis. By restricting contemporaneous responses and tracing the delayed adjustment of consumption, we see clear evidence that:

1. Income shocks have an immediate effect on output (as expected),

2. Consumption responds only over time — not on impact — confirming rational expectations and consumption smoothing behavior.

The use of a short-run identified SVAR thus allows us to causally interpret income shocks and assess their dynamic impact on consumption and output.